Finnish elementary teachers' conceptions on problem solving in mathematics teaching

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Abstract. *According to the curriculum (NBE, 2004), improving pupils' problem solving skills is an important objective. Teachers' role is crucial in carrying out the objectives of the curriculum. Especially their conceptions influence decisions they make when teaching mathematics. The purpose of this paper is to figure out what kind of conceptions elementary teachers have concerning problem solving and its teaching in mathematics. The data was gathered with a questionnaire consisting of open questions during springs 2006 and 2007. All the elementary teachers (grades 1–6) of the city Kerava (N = 103) in the southern Finland was given the questionnaire, but the responding rate was only 41%. According to teachers, problem solving in mathematics means various problems, strategies, mathematics in everyday situations, pupils' own thinking and applying previously learned skills. In the teachers' conceptions of teaching problem solving in mathematics, concrete and practical approaches are emphasized.*

Keywords: elementary teacher, conception, problem solving, mathematics

Sunto. *Secondo il curricolo (NBE, 2004), migliorare le abilità degli studenti nel problem solving è un obiettivo importante. Il ruolo degli insegnanti è fondamentale per realizzare gli obiettivi del curricolo. Soprattutto, le loro concezioni influenzano le decisioni che prendono quando insegnano matematica. Lo scopo di questo lavoro è quello di capire che tipo di concezioni hanno gli insegnanti elementari riguardo al problem solving e al suo insegnamento in matematica. I dati sono stati raccolti con un questionario composto di domande aperte durante le primavere 2006 e il 2007. Tutti gli insegnanti elementari (dalla prima alla sesta) della città di Kerava (in numero di 103) nel sud della Finlandia sono stati sottoposti al test, ma solo il 41% ha dato risposta. Secondo gli insegnanti, il problem solving matematico comporta vari problemi, strategie, matematica delle situazioni quotidiane, pensiero personale degli studenti e applicazione delle abilità precedentemente apprese. Nelle concezioni dei docenti sull'insegnamento del problem solving in matematica, sono enfatizzati approcci concreti e pratici.*

Parole chiave: insegnante di primaria, concezione, problem solving, matematica

Resumen. *Según el currículo (NBE, 2004), mejorar las habilidades de los estudiantes en la actividad de problem solving es un objetivo importante. El papel de los docentes es fundamental para alcanzar los objetivos del currículo. En particular, sus concepciones influyen en las decisiones que toman cuando enseñan matemática. El objetivo de este trabajo es el de entender qué tipo de concepciones tienen los docentes de la escuela primaria en relación con el problem solving y de su utilización en la enseñanza en matemática. Los datos se recogieron con un cuestionario compuesto por preguntas abiertas durante abril y mayo de 2006 y de 2007. Todos los*

docentes de primaria (de primero a sexto) de la ciudad de Kerava (un total de 103 docentes) en el sur de Finlandia se sometieron a un test, pero sólo el 41% lo respondió. Según los docentes, el problem solving matemático implica varios problemas, estrategias, matemática de las situaciones cotidianas, pensamiento personal del estudiante y aplicaciones de las habilidades precedentemente adquiridas. En las concepciones de los docentes sobre la enseñanza del problem solving en matemática, se enfatizan una dirección concreta y práctica.

Palabras clave: docente de primaria, concepciones, problem solving, matemática

1. Introduction

In Finland we have a nine-year comprehensive school where all children learn in heterogeneous classes. The class size varies between 20–30 pupils, and therefore teachers have difficulties in balancing between low-attainers and successful pupils, especially in upper grades (grades 7–9). After the comprehensive school, about half of the age cohort selects to continue in upper secondary school (3–4 years). In mathematics, there are two options for students to select: general mathematics and advanced mathematics. After the upper secondary school, there is the matriculation examination where mathematics is optional; about one third of all students take mathematics. See more on the Finnish school system in the published book Pehkonen, Ahtee, and Lavonen (2007).

To mathematical problem solving it is given much emphasis in the planning of Finnish basic education. This can be seen, among others, in the basics of the recent national curriculum for the comprehensive school (NBE, 2004). In the basics given to schools by the National Board of Education, it is defined that mathematics teaching in the comprehensive school delivers to pupils mastering of mathematical concepts and the most common solution methods in basic mathematics. Another objective of teaching is that it should develop pupils' mathematical thinking, and conduct pupils to find, elaborate and solve problems. Mathematics teaching influences pupils' spiritual growth and teaches purposeful performance. In the descriptions of key contents and good achievement for different grades, it is repeated the emphases of mathematical problem solving and thinking skills, from even the very first years of school (ibid).

2. Theoretical background

2.1. Problem solving in mathematics teaching

Problem solving has generally been accepted as a mean for advancing thinking skills in school (e.g. Schoenfeld, 1985). And this objective can be read also in the Finnish curriculum (NBE, 2004). But the basic concepts, 'problem' and

'problem solving' seem still to be rather ambiguous in mathematics education. Sometimes a 'problem' is understood to be a simple arithmetic task that can be solved in a routine way, whereas at other times it means a more complex situation. The fuzziness of problem solving concepts is discussed e.g. in Pehkonen (2001).

The nature of problem solving has been described in the literature with the help of problem solving models (e.g. Mason, Burton, & Stacey, 1985; Polya, 1945; Schoenfeld, 1985). Polya's four-step model that is today about 60-year-old might be the most common one: Understanding the problem, Devising a plan, Carrying out the plan, Looking back (Polya, 1945). The problems in Polya's model have been the oversimplified structure of the model, the model looks like a receipt. But real problem solving is not possible via following such a scheme, the solver needs to use his/her own creativity. Therefore, the model is modified by other researchers, usually by refining some step/ steps. One of these refinements is the model that Schoenfeld has used. After him the problem solving process goes through the following five stages: Read, Analyze, Explore, Plan, Implement, Verify (Schoenfeld, 1985).

Another point of critics is that most of these models are linear, and in reality exploring new mathematics (i.e. problem solving) is not linear, not so systematic. Until Mason et al. (1985) have developed such a model for problem solving that corresponds the reality of mathematical exploration. The model has only three phases: Entry, Attack, Review, since the authors have combined the two middle steps in Polya's model. But their key idea is that between the phases Entry and Attack there is a mulling circle in the following sense: In the Entry phase, the solver gets a possible solution idea (AHA!), and he/she tries to follow it as far as possible. But earlier or later he/she will get STUCK!, and is compelled to fold back to the Entry phase. The mulling in this circle will end until when the solver finds a correct way out, i.e. when he/she solves the problem (Mason et al., 1985, p. 131).

Another way of considering the mulling circle is the way Kiesswetter deals it with the elementary graph theory (Kiesswetter, 1983): He speaks about a solver's material graph (the knowledge structure) on the problem that he/she develops. Mulling from AHA to STUCK and back means that the solver enriches his/her material graph, finding new connections between the previous facts. This continues as long as the solver can see the solution in his/her material graph, thus he/she has developed a solution graph, i.e. solved the problem.

Mason's interpretation of problem solving is compatible to constructivist understanding of learning (e.g. Davis, Maher, & Noddings, 1990). One promising method for mathematics teaching seems to be the so-called "open approach". In that teaching conception, the teacher offers his/her class an open learning environment, in the form of an open-ended problem (Pehkonen, 2001). His/her aim is to develop pupils' mathematical problem-solving skills,

and to give pupils an opportunity to learn on their own way and at their own rate. For this purpose we can use open tasks that have been accepted as a promising solution in order to create a proper learning environment.

Teaching problem solving is challenging for a teacher in three different ways (Burkhardt, 1988): It is *mathematically* challenging, since he/she must understand and master the mathematics needed as well as the properties of pupils' different solution models. It is *pedagogically* challenging, since the teacher should decide when to let a pupil continue and when to stop his/her working as well as to know what kind of advices and hints will help the pupil further. As the third point, it is challenging on a *personal level*, since in problem solving it might occur such a situation, where the teacher does not even him/herself know how to continue, and to meet such a situation is to most people a very painful feeling.

2.2. Teachers' mathematics-related conceptions

In earlier studies, it has been noticed that teachers' conceptions are of paramount importance when trying to understand teaching situation (e.g. Cooney, 1985, 1988; Grouws, Good, & Dougherty, 1990; Thompson, 1988). Teachers' conceptions develop through their own experiences from teaching. Naturally teacher pre-service and in-service education forms also their understanding on teaching problem solving.

There is a big variety of answers to the question "What is mathematics?" which hints that there is not only one understanding of mathematics, but several different views of mathematics. And not in the sense that there is only one *right* view of mathematics and the others are *wrong*. Philosophers of mathematics (e.g. Hersh, 1997), and mathematics education (e.g. Ernest, 1998) have introduced several *right* views of mathematics that are also accepted among mathematicians. The same is valid also in problem solving: everybody has his/her own understanding of problem solving that was clearly shown e.g. in the study of Stecher and Mitchell (1995). Therefore, the study on teachers' conceptions and their development is important.

But conception is problematic as a concept, since it is connected to many neighbourhood concepts, as belief, view, attitude, knowledge, and they are not clearly defined (Furinghetti & Pehkonen, 2002). The following characterization is used in the literature (Pehkonen & Hannula, 2004): An individual's *beliefs* are understood in a rather wide sense as his/her subjective, experience-based, often implicit knowledge and emotions on some matter or state of art. Furthermore, we explain conceptions as conscious beliefs. In the case of conceptions, we understand that the cognitive component of beliefs is stressed, whereas in basic (primitive) beliefs the affective component is emphasized.

School experiences have a remarkable role in the birth of mathematics-related beliefs and conceptions. Research results have revealed an alarming

devil's circle: It seems that teachers will teach, as they have been taught. And their pupils will continue as future teachers in the same devil's circle. Teachers will select teaching topics and make decisions on the organization of teaching based on their beliefs, conceptions and attitudes about mathematics and it's teaching. Such beliefs, conceptions and attitudes are based on their own school-time experiences on mathematics (Lindgren, 1996).

2.3. Earlier research on teachers' conceptions of problem solving

Within last 20 years, there are published several studies on teachers' conceptions of problem solving. Here we discuss briefly six of them, as examples. Burns and Lash (1988) examined how teachers' conceptions about teaching mathematics influence the manner in which they plan instruction in mathematical problem solving. Results demonstrated that teachers had a limited knowledge of teaching techniques and that teachers' concerns focused more on collection of materials and resources than on how to teach problem solving.

Grouws et al. (1990) interviewed 25 lower secondary school teachers concerning their beliefs and teaching practices; especially problem solving was emphasized. Teachers' definitions for problem solving could be classified into four groups: Problem solving means (1) verbal tasks, (2) finding solutions for tasks, (3) solving practical tasks, (4) solving tasks demanding thinking.

Pehkonen (1993) investigated Finnish teacher educators' conceptions on the implementation of problem solving. The data was gathered with a questionnaire from 43 teacher educators in a Problem Solving seminar. The results could be suppressed into four points: Problem solving is important, since it helps the fostering of pupils' cognitive readiness. Teaching problem solving should be carried out in a creative, flexible and approving manner. Teachers should involve pupils in problem solving through letting them solve their own problems. Pupils' readiness to study problem solving was considered the most important prerequisite for teaching problem solving.

Stecher and Mitchell (1995) examined the impact of portfolio assessment program by exploring 20 fourth-grade teachers' conceptions of problem solving. Teachers indicated that program has enhanced their understanding of mathematical problem solving and broadened their instructional practices. At the same time they admit that they have encountered difficulty in understanding certain components of the reform and making relevant changes.
Verschaffel, De Corte, and Borghart (1997) administered fourteen word problems, half of which were problematic from a realistic point of view, to 332 Belgian pre-service elementary school teachers who also saw answers given by four students. Results revealed a strong tendency to exclude real-world knowledge from spontaneous solutions and appreciations of student-supplied answers.

Pehkonen (1999) dealt with in-service teachers' conceptions on open tasks.

Lower secondary teachers (N=74), selected at random, answered a postal survey (the answering percentage was about 50%) inquiring their knowledge about open tasks. The results can be summarized, as follows: About a half of test subjects were not acquainted with the concept "open task". There are good reasons to believe that the non-respondents did not know the concept. Therefore, one may conclude that approximately only one quarter of the Finnish lower secondary school teachers is familiar with the term "open task".

2.4. Focus of the paper

In conformity with the curriculum (NBE, 2004), improving pupils' problem solving skills is important. Teachers' role is crucial in carrying out the objectives of the curriculum. Especially their conceptions influence decisions they make when preparing their lessons and when teaching mathematics. The aim of this paper is to figure out: What kind of conceptions Finnish elementary in-service teachers have concerning problem solving and it's teaching in mathematics?

3. Methods

A questionnaire with open questions was given to all elementary in-service teachers (grades 1–6) of the city of Kerava during the springs 2006 and 2007. Kerava is a small city in southern Finland, about 30 km to the north from Helsinki. There are 103 elementary teachers in Kerava, but we received responses only from 42 of them. In order to improve the response rate different methods were used, e.g. the questionnaire was sent again to elementary teachers in Kerava, but it had no significant result. It resulted only a couple of more replies. Thus, the reply rate was unfortunately as low as 41%. The paper at hand is mainly based on the unpublished master's thesis (Sivunen, 2007).

The questionnaire used was constructed especially for the study. It contained of six open questions, and usual closed background questions, concerning gender, age, qualification, specialization in mathematics, and teaching experience. The open questions were, as follows:

- What does problem solving mean for you in mathematics teaching?
- Why is problem solving a part of the mathematics curriculum according to your view?
- How should problem solving be taught in mathematics?
- How can you see problem solving in your mathematics teaching?
- What kind of facilities do you have to teach problem solving in mathematics?
- What kind of obstacles have you experienced when teaching problem solving in mathematics?

On the one hand since the group of the respondents was so small, teachers were not divided into sub-groups according to the grade they teach this year, i.e. the background questions were not used. On the other hand Finnish teachers are usually moving from one grade to another with their pupils; therefore, they cannot be labelled as a teacher of one certain grade. The method of content analysis was used in analyzing the research material.

4. On results and their interpretation

From the teachers' responses, their conceptions on problem solving and its teaching could be classified with the method of content analysis into three groups: (1) on the meaning of the curriculum, (2) on the meaning of teaching materials, (3) on teaching of problem solving skills.

4.1. On the meaning of the curriculum

The purpose of mathematics teaching to support through problem solving pupils' development in data dealing and elaborating (NBE, 2004) is very up-to-date. The emphasis in teaching is set in mastering of mathematical calculations and concepts. And teaching of problem solving skills is totally left away from the key contents of mathematics before grade 6 in the comprehensive school. They will be assessed, but there is no framework to teach them. As if school administration thinks that they will be developed automatically when working with mathematical routine tasks. Development in problem solving demands much time from pupils.

The following quotes from the teachers' responses emphasize the meaning of curriculum:

> The only problem is the limitedness of time. In the curriculum there are too many topics, in order we had time enough to concentrate on problem solving.
>
> The lack of time. In the curriculum there are so many topics to be taught that problem tasks seem to have less time, if one will not give special attention to it.
>
> Hurry. Too many basic topics to be taught. Low-attainers need time and support in basic topics.
>
> The lack of time. The requirements of the curriculum behind one's neck, a teacher has not always courage to stop and give children time to think.

The problematic situation described above can be seen also in the teachers' conceptions on teaching problem solving. In line with the teachers' conceptions, there are so many content topics written in the curriculum that there does not seem to be enough time for teaching problem solving. Here we can conclude that the teachers define mathematical topics to be taught just via the material objectives written in the curriculum and named "key topics". Thus they leave the formal objectives where problem solving is written also in the

case of lower grades, for less attention when planning their instruction. In relation to conceptions of the teachers participating in the study, there is not enough time for problem solving, since there are many so-called "basic topics" in mathematics teaching that pupils should rule. Here they think that learning of mathematical "basic skills" is in the objectives hierarchically higher than problem solving skills. Teachers seem to consider these two skills as separated from each other.

Based on this conception it is very understandable that there is too little time for problem solving in mathematics curriculum and instruction, and consequently only brilliant pupils are working with problem solving. There is a danger that low-attainers in mathematics will work mainly with basic routine tasks every year. The teachers told that they use in mathematics problem-solving tasks as additional tasks with which they differentiate their teaching, especially in the case of talented and motivated pupils.

On the one hand, the teachers participating the study expressed the conception that problem solving means the application of earlier learned mathematical knowledge and skills. This could be interpreted that according to the teachers' conceptions pupils should rule calculations before they use them in problem solving. Similar thinking is also reflected from the curriculum (NBE, 2004). On the other hand, the teachers expressed that teaching problem solving needs much time according to their understanding. The teachers seem to be in a very embarrassing situation. They see the importance of problem solving as a part of mathematics teaching, but they are not able to implement such a teaching that corresponds their conceptions because of outer pressure, as those from the curriculum, since there are many content objectives and not enough time. To this social context belongs also the curriculum used that seems to be in contradiction with the teachers' conceptions when the focus is the material objectives of the curriculum.

If problem solving is understood to be a part of mathematics teaching, its position is not a separate one, but it will have a very central position as a teaching method and as a content to be learned. Such a problem-centered approach is also written in the curriculum (NBE, 2004). The aim is to approach problem-centered topics to be taught in mathematics. In the data at hand, there is not to be seen such a teacher conception on problem solving. Only two respondents (from 43 teachers) mentioned sometimes to use problem solving when approaching a new topic to be learned.

4.2. On the meaning of teaching materials

Teaching materials raised often in the analysis of the data. When the teachers participating the study were asked what they would mean with problem solving, the main idea that emerged was that problem solving means certain kind of tasks. The teacher described these tasks with some characteristics features. In conformity with the teachers, problem-solving tasks demand from

pupils independent creative thinking, reasoning and applying. Tasks can be in verbal or visual form, and they should be new for pupils. Tasks used should also be connected with practical everyday situations.

The following comments from the teachers' responses introduce the meaning of teaching materials:

> Already a verbal task is problem solving. Different tasks that require many-sided creative thinking. The mere mechanical mastering of topics is not enough, application is required.
>
> Application of learned skills.
>
> Verbal, picture, non-mechanical tasks, all they train independent thinking.
>
> Application tasks. Tasks where one is compelled to apply a learned topic in a new situation.
>
> I use the material of textbook authors, as different verbal, picture-puzzle tasks. Traditional verbal tasks are tried to solve in the way that different (several) solutions are pondered.
>
> One is compelled to collect tasks, if he/she is not willing to stick in the tasks of the textbook. It is rather tedious.

Most of the teachers participating the study expressed their similar worry on the state of mathematical teaching materials. They were experienced to contain too little material proper for teaching problem solving. However, most of the teachers responding the questionnaire told that they use in mathematics teaching much textbook and its teachers' guidebook. The teachers had the feeling that especially the teachers' guidebook helped them in teaching problem solving. Although some of the teachers told that they use material outside of the textbook, the use of textbook was clearly emphasized in the data. The teachers spoke about verbal tasks. In line with the teachers' conceptions, these tasks can be used in teaching problem solving when different solution alternatives are discussed with pupils. These problems are, however, closed in their nature, and therefore, there is a very limited amount of solution methods. The teachers also told that they use in problem solving mathematical puzzles and pondering problems presented in different verbal or visual form, as well as mathematical learning games.

In relation to the teachers' conceptions, teaching material was experienced as a hindering factor for teaching problem solving. Research results show that the teachers use in their teaching material that suits poorly for teaching problem solving according to their conceptions. This could be explained with the teachers' statement that searching, gathering and producing proper material for teaching problem solving is tedious work and demands time. The teachers implement well problems connected with teaching materials, but they are not able to fix them according to their own understanding.

4.3. Teaching of problem solving skills

Teaching problem solving skills is not contained into the material objectives of mathematics curriculum for lower grades (grades 1–6) of the comprehensive school (NBE, 2004). In the responses of the teachers participating to the study, one can see that problem solving in mathematics teaching means the use and studying of different strategies. According to the teachers, strategies are needed in solving problems. A pupil selects and combines proper strategies with his/her logical thinking for solving a problem.

The following quotes from the teachers' responses in the questionnaire stress the teaching of problem solving skills:

> To study strategies with which the solver can solve new types of tasks, i.e. problems.
>
> A pupil selects him/herself what strategy he/she is using. There is not always necessarily a 'ready formula' taught what the pupil can use.
>
> A teacher can model and teach step-by-step thinking in stages, i.e. to teach thinking strategies: to illustrate with different models, blocks, etc.

In the teachers' conceptions on how problem solving should be taught in mathematics, there is also an idea that a teacher should act as a leader for problem pondering who illustrates, gives examples and opens problems. The teacher should also explain his/her own thinking during the solution process to his/her pupils, as well as give them information on reason-cause-relationships and on different problem solving strategies. The teachers mention here such strategies as advancing of step-by-step and dividing the problem into sub-problems. The teachers speak also about a process-oriented approach that in conformity with their conceptions suits well to teaching problem solving. In that case the teacher acts as a guide for the process. In line with the teachers' conceptions, their task is also to select problems to be dealt with.

The teachers' conceptions connected with teaching problem solving reflect also their conceptions on their own teaching of problem solving. In relation to the teachers' conceptions, teaching problem solving skills can be seen in their instruction as pondering of different solution alternatives. With this they mean the method they used in solving verbal mathematical problems, where the class under the teacher's guidance ponders possible solution procedures for the problem at hand.

Teaching problem solving skills is, however, not emphasized in the teachers' conceptions on problem solving in mathematics teaching. Only a small group of teachers mentioned problem-solving skills in some form or other in their conceptions. Based on the data one may conclude that a part of the teachers experiences that teaching and practicing problem solving skills belongs to mathematics teaching, but they do not express any established practices how they teach these skills. The only practical hint in the data is the pondering of different solution alternatives when solving verbal tasks.

Similarly pupils' own thinking and their thinking skills are understood to be a part of the nature of problem solving. In the teachers' conceptions on their own teaching of problem solving, it is emphasized instead of problem solving skills the teaching materials used and their tasks, as well as the time their pupils use for problem solving during mathematics lessons.

5. Discussion

According to the teachers, problem solving in mathematics means various problems, strategies, mathematics in everyday situations, pupils' own thinking and applying previously learned skills. In the teachers' conceptions of teaching problem solving in mathematics, concrete and practical approaches are stressed. However, in the teachers' own mathematics instruction the significance of teaching materials was emphasized.

Concerning their conceptions about the resources to teach problem solving, the teachers established the importance of education, experience and teaching materials. In the teachers' conceptions, it was stressed as obstacles for teaching problem solving when there are insufficient time to teach problem solving as well as pupils' poor skills and resources. Similar obstacles were seen also in the questionnaire responses of the Finnish teacher educators about fifteen years ago (Pehkonen, 1993). Additionally problem solving is not in the core content of mathematics in lower elementary grades, and therefore, the teachers feel that there is not enough time to teach it. Curricula, teaching materials and the teachers seem to emphasize pupils' basic calculation skills more than their problem solving skills, and they seem to consider these as separate contents.

In the data of the study, one can read the pedagogical challenge of teaching problem solving for a teacher (Burkhardt, 1988). The teachers told how they balance with their large teaching groups that contain very different-level pupils of their knowledge, skills and learning abilities. Some pupils are interested in mathematics and well motivated to work with problem solving. Whereas some other pupils experience even mathematics to be difficult, and might refuse fully to work with tasks if they do not immediately get guidance for the correct way to solve and to the solution. On the one hand, the teachers put forward also the insufficiency occurring at times of their own skills in these challenging situations. It was felt often difficult to give correct hints and advices to different pupils in due time. A part of the teachers pondered how to guide a pupil in such a way that the true solving was left to the pupil self. On the other hand, there are also some teachers in the group of the respondents whose experience and own interest in problem solving had helped them forward in finding own teaching methods and proper materials.

Kush and Ball (1986) displayed a classification of teachers' conceptions on mathematics teaching and learning. In the teachers' conceptions of the study,

there are elements of content-oriented conceptions given in this classification. Kush and Ball divide these conceptions into two groups: those emphasizing understanding and those emphasizing calculation skills. In the data of the study at hand, it is not possible to do a covering and reliable analysis on the teachers' conceptions in relation to the classification of Kush and Ball (1986), because the group of respondents with conceptions on a teacher's role is very small. Generally it can be stated that in the teachers' conceptions on their own role in teaching problem solving, there are mainly teacher-centred teaching methods. Only two respondents of 43 mentioned teaching problem solving with a process-oriented approach.

It is interesting to notice that teachers' conceptions of problem solving have not much developed in the time slot of 20 years: Burns and Lash (1988) reported that teachers had a limited knowledge of teaching techniques and that teachers' concerns focused on collection of materials. Also Grouws et al. (1990) singled out that teachers' interests in problem solving means mainly verbal tasks and their collection. Those results are in line with our findings. Similar results came also from Finnish teacher educators (Pehkonen, 1993) and from Finnish teachers of the third-grade (Näveri el al., 2011). Thus, teachers' conceptions seem to change very slowly, if at all.

5.1. Concluding note

Some of the teachers in the study expressed that they need in-service education in teaching problem solving. In the data, the emphasis was remarkably in the meaning of in-service education for teachers' ability to teach problem solving. Some teachers told also that their teacher education has not provided enough means to teach problem solving, or these means were not sufficient. Similar concerns were expressed about problem solving also earlier by the Finnish teacher educators (Pehkonen, 1993).

The teachers' conceptions on problem solving belongs to a larger totality that includes at least teachers' conceptions on the nature of mathematics (e.g. Ernest, 1998), its teaching (e.g. Kush & Ball, 1986) and learning. Therefore, the change of teachers' conceptions on problem solving is a large process that demands before all a teacher's own reflective thinking (Thompson, 1984). In the published paper (Pehkonen, 2006), it is pointed out that such a conceptual change might also be a radical one (e.g. Merenluoto, 2005), and therefore, the most complicated one.

Although the development in teaching problem solving in Finland has not been as rapid as expected, there are some changes to be observed (Pehkonen, Hannula, & Björkqvist, 2007). The use of problem solving tasks is quite popular today in Finnish mathematics lessons, but mainly in the form of mathematical puzzles. If we use the language introduced by Schroeder and Lester (1989), we might say that only very few teachers are teaching *via* problem solving, while most of them teach something *about* problem solving.

The latter means that they might use some mathematical puzzles in their teaching or have a problem box in their class or something similar. And the former states that these teachers use problem solving as a teaching method, and that is very rare.

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